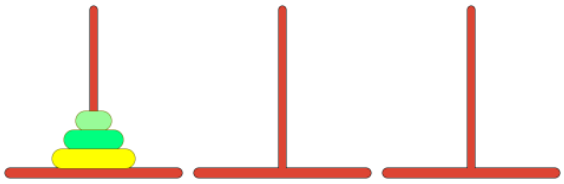
The Towers of Hanoi - Key

A monastery in Hanoi has a golden board with three wooden pegs on it. The first of the pegs holds 64 gold disks in descending order of size—the largest at the bottom, the smallest at the top. The monks have to move all the disks to the third peg while keeping them in descending order, one at a time. A larger disk must never sit on a smaller one. All three pegs can be used. When the monks move the last disk, the world will end. Should we be worried? How long will this take?

Try the puzzle yourself with disks (coins) of different sizes.

THERE ARE ONLY 2 RULES:

1. You can only move one disk at a time, and
2. You cannot ever put a larger disk on a smaller disk



What is the minimum number of moves needed to complete the Tower of Hanoi game with:
i) One disc? **1**
ii) Two discs? **3**
iii) Three discs? **7**
iv) Four discs? **15**

Do you notice anything interesting about the way the number of moves increases?
Can you explain any patterns you find?

Can you use the minimum number of moves to work out how many moves would be needed with five discs?

In general, can you describe a way of working out how many moves are needed when one extra disc is added?

n=number of discs

M=number of moves

n	M
1	1
2	3
3	7
4	15

Stack of 2 needs to be moved to the middle before the largest disk can be moved to the third peg. It takes 3 moves total to get top 2 disks in the middle. It then will take 1 move to get the largest disk to the last peg, and 3 moves to get the remaining disks to the third peg: Moves = 3+1+3=7 =

$M_{n-1}+1+M_{n-1}=2M_{n-1}$

$M_n=2M_{n-1}+1$

Explain how you could work out the number of moves needed for the Tower of Hanoi puzzle with *n* discs.
n=number of discs

M=number of moves

$M=2^n-1$

When the final move is made in the temple, the world will come to an end. If one move is made each second, how long would it take to complete the game with 64 discs?

2^n-1

2^{64}-1 = 590 billion years

60x60x24x365 = 31,636,000 sec/years